GENERATION OF TEMPERATURE COMPENSATED VOLTAGES

by

ELDON LEE MICKELSON

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Approved by:

Norbert R. Malik
Major Professor

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INTRODUCTION

The adverse effects of changing temperatures on electrical network performance is of increasing importance. Circuits of high linearity and stability are desired, yet they must often operate under extreme temperature variations. Designers may attempt to limit adverse temperature effects in one of two ways. First, the circuit may be designed in such a way that variations in temperature have as little effect as possible. This is usually only a partial solution and is costly in terms of desired circuit performance. Second, additional circuitry may be designed to compensate for the undesired temperature effects. For example the gain of a feedback amplifier may vary with temperature. This variation may be partially eliminated by increasing the feedback, but the gain is decreased in the process. The temperature induced gain variation may be eliminated by making the feedback voltage vary with temperature in the proper manner. The frequency-temperature characteristic of a crystal resonator may be linearized by varying the crystal load capacitance. This capacitance may be provided by a varactor biased by the proper d.c. control voltage. In both of these examples as well as in other compensation problems, generation of a temperature variable control voltage is required.

The subject of this study is the design of a circuit to provide a specified temperature compensated control voltage. The possibility of approximating a thermistor by the first few

terms of a power series was studied. This did not yield useful results. A linear approximation was made for a thermistor and resistor in parallel. This linear approximation was used in the study of a voltage divider and a bridge network. A method of generating temperature compensated voltages using the bridge network was devised. This generation method was applied to the problem of compensating a crystal resonator.

DESIGNING A TEMPERATURE COMPENSATING CIRCUIT

The design of a temperature compensation circuit requires a device whose physical properties vary with temperature. Devices with temperature variable resistance, capacitance, or inductance are available or could be constructed. Certain properties are necessary or desirable in such a device. The thermistor satisfies most of them. The range of resistance variation with temperature is large. Thermistor resistance can vary by a factor of 10^5 over a 200° C temperature range. The thermistor is also inexpensive. The effect of the thermistor on the compensated circuit or device can be easily modified by placing it in various resistance networks. If a device were compensated by means of a temperature variable capacitance or inductance, the effect of the compensating element would have to be modified by capacitive or inductive networks. The resistive network necessary for a thermistor would be less expensive than an inductive or capacitive network.

Thermistor Characteristics

Thermistors are semiconductor devices. Their resistivity lies between that of conductors and insulators. Thermistors are made from both N-type and P-type semiconductor materials, generally mixtures of manganese, nickel, and cobalt oxides. Conduction in either type of material is the result of the generation of intrinsic electron-hole pairs. These are formed by thermally

excited electrons being elevated to the conduction band. At higher temperatures more electron-hole pairs are formed and hence there is higher conduction and lower resistance. The resistance of a thermistor is given by

$$R_t = R_o e^{B(^1/_T - ^1/_{T_o})} \quad , \tag{1}$$

where R_o is the resistance at some nominal temperature T_o, T is the temperature of interest, and B ia a constant for a particular thermistor. Depending on thermistor materials and construction R_o may vary from a low of 100 ohms or leas to a high of several megohms. Not strictly a constant, B may vary slightly with temperature but may be considered constant over any 150° C temperature range. Thermistor materials display a wide range of B values, varying from a low of almost zero to a high of about 7,000. The units of B are degrees centigrade or Kelvin. This wide range of thermistor constanta makes poasible the selection of a thermistor to suit the needs of a particular compensation problem. The thermistor temperature coefficient of resistance, which may be derived from Eq. (1) by evaluating $\frac{dR_t/dT}{R_t}$, is

$$\alpha = {}^{-B}/_{T^2} \quad . \tag{2}$$

The temperature coefficient of resistance is negative as would be expected of a semiconductor device and is highly nonlinear with respect to temperature.

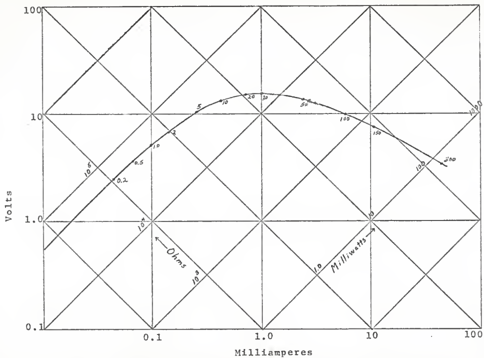


Fig. 1 Static voltage-current plot for a .061 centimeter diameter bead of thermistor material suspended in air, B = 3,920° K.

When placed in a practical circuit, a thermistor will have some voltage across it. A current will flow causing power to be dissipated in the thermistor. This power dissipation will result in an increase in the thermistor temperature and consequently a decrease in resistance. A logarithmic plot of a static voltage-current curve is given in Fig. 1. The numbers along the curve give the rise in temperature above ambient. A current of less than 0.2 milliamperes, resulting in less than a 2 degree temperature rise has little effect on the resistance of this thermistor. The dissipation constant of a thermistor, C, is

$$C = {}^{P}/_{\Delta T} \quad , \tag{3}$$

where P is the number of watts of power dissipated by the thermistor and ΔT is the temperature rise above ambient. A quantitative measure of the effect of heating on thermistor resistance is the power sensitivity. The power sensitivity is the number of watts of power which may be dissipated before the thermistor resistance decreases by one percent. The power sensitivity, S, is expressed in terms of the power dissipation constant and the temperature coefficient of resistance by

$$S = \frac{C}{\alpha \times 100} \quad . \tag{4}$$

The dissipation constant is affected by the surface area and the volume of the thermistor, the nature of the surface, and the thermal conductivity of the surroundings and supports. The

dissipation constant may be altered by varying the physical size and shape of the thermistor, however this may also affect the value of R_o. The time necessary for the thermistor to respond to a change in ambient temperature is inversely proportional to the dissipation constant. An average value for C is 5 milliwatts per degree centigrade, however this is variable by a factor of 10 or greater.

When used for a temperature compensation element, the temperature and hence the resistance of the thermistor must be determined by ambient conditions and not internal heating. A necessary restriction for such compensation circuits is that thermistor currents be limited to low levels. Values of currents well within those dictated by the rated power sensitivity should be used. Since ambient conditions may change rapidly, the dissipation constant should be chosen so that the compensating thermistors and the compensated elements both respond to a change in ambient conditions at approximately the same rate. If this is not done, sudden changes in ambient conditions will cause transient periods of under or over compensation. Since both C and R_o are a function of thermistor size, it might be impossible to satisfy specified restrictions on both parameters in a single thermistor. This problem might be eliminated by using the desired value of C to determine the thermistor type and by placing two or more units in series or parallel to obtain the desired equivalent value of R_o. Finally, it is important that the compensated elements and the compensating thermistors be placed in close physical proximity,

Fig. 2 A hypothetical temperature variable compensation voltage.

since the temperature difference possible between two network points is not necessarily constant in time. For example, a sudden temperature increase in the vicinity of a certain resistor caused by an increase in load current would not immediately result in a temperature increase at a more distant point in the network.

Several temperature compensation methods require the use of a d.c. control voltage whose amplitude varies with temperature. Such a voltage is illustrated in Fig. 2, where temperatures T_a and T_b are the end points of the range of desired compensation. Often this voltage is given or may be accurately approximated over a desired temperature range by a polynomial f(T). Such a control voltage may be realized by a resistive network having a temperature variable transfer function, K(T), driven by a constant voltage source. The transfer function may in general be approximated by a ratio of polynomials such that,

$$K(T) = \frac{N(T)}{D(T)} \quad . \tag{5}$$

This network would include thermistors as the temperature variable elements and would hopefully be such that K(T)xV, V being the source voltage, is a close approximation of f(T) over the limited range of interest, T_a to T_b.

Power Series Representation of a Thermistor

Equation (1) may be written as a power series in T expanded about some temperature $T = q$. It is quite possible that the first few terms of the power series might give an accurate approximation of R_t over the temperature range T_a to T_b. The power series representation of the thermistor might then be used in the transfer function equations of a network. This transfer function is then of the form K(T). The known coefficients of f(T) and the coefficients of the power series provide sufficient information to determine the unknown coefficients of K(T). The coefficients of K(T) would then be used to find the values of the network resistors which are not temperature variant.

To determine the accuracy of the power series representation of R_t, the power series was computed using the first seven terms. Restating Eq. (1),

$$R_t = R_o \, e^{-B/T_o} \, e^{B/T} = k \, e^{B/T} \quad . \tag{6}$$

The power series expansion at $T = q$ is

$$R_t = d_o + \frac{d_1 \times (T-q)}{1!} + \frac{d_2 \times (T-q)^2}{2!} + \cdots + \frac{d_6 \times (T-q)^6}{6!} + \ldots \tag{7}$$

Each d_i of Eq. (7) denotes the i^{th} derivative of Eq. (6) evaluated at $T = q$. As listed below the first seven are

$$d_o = k\, e^{B/q} \quad ,$$

$$d_1 = -\frac{k\ B}{q^2}\, e^{B/q} \quad ,$$

$$d_2 = \frac{2\ k\ B}{q^3}\, e^{B/q} + \frac{k\ B^2}{q^4}\, e^{B/q} \quad ,$$

$$d_3 = -\frac{6\ k\ B}{q^4}\, e^{B/q} - \frac{6\ k\ B^2}{q^5}\, e^{B/q} - \frac{k\ B^3}{q^6}\, e^{B/q}$$

$$d_4 = \frac{24\ k\ B}{q^5}\, e^{B/q} + \frac{36\ k\ B^2}{q^6}\, e^{B/q} + \frac{12\ k\ B^3}{q^7}\, e^{B/q} + \frac{k\ B^4}{q^8}\, e^{B/q} \quad ,$$

$$d_5 = \frac{-120\ k\ B}{q^6}\, e^{B/q} - \frac{240\ k\ B^2}{q^7}\, e^{B/q} - \frac{120\ k\ B^3}{q^8}\, e^{B/q} - \frac{20\ k\ B^4}{q^9}\, e^{B/q}$$
$$- \frac{k\ B^5}{q^{10}}\, e^{B/q} \quad ,$$

$$d_6 = \frac{720\ k\ B}{q^7}\, e^{B/q} + \frac{1800\ k\ B^2}{q^8}\, e^{B/q} + \frac{1200\ k\ B^3}{q^9}\, e^{B/q}$$
$$+ \frac{300\ k\ B^4}{q^{10}}\, e^{B/q} + \frac{30\ k\ B^5}{q^{11}}\, e^{B/q} + \frac{k\ B^6}{q^{12}}\, e^{B/q} \quad . \qquad (8)$$

The power series for R_t may therefore be written as

$$R_t = (d_o - d_1 q + \frac{d_2 q^2}{2} - \frac{d_3 q^3}{6} + \frac{d_4 q^4}{24} - \frac{d_5 q^5}{120} + \frac{d_6 q^6}{720})$$
$$+ (d_1 - \frac{2 d_2 q}{2} + \frac{3 d_3 q^2}{6} - \frac{4 d_4 q^3}{24} + \frac{5 d_5 q^4}{120} - \frac{6 d_6 q^5}{720}) \times T$$
$$+ (d_2 - \frac{3 d_3 q}{6} + \frac{6 d_4 q^2}{24} - \frac{10 d_5 q^3}{120} + \frac{15 d_6 q^4}{720}) \times T^2$$

$$+ \left(\frac{d_3}{6} - \frac{r d_4 q}{24} + \frac{10 d_5 q^2}{120} - \frac{20 d_6 q^3}{720}\right) \times T^3$$

$$+ \left(\frac{d_4}{24} - \frac{5 d_5 q}{120} + \frac{15 d_6 q^2}{720}\right) \times T^4$$

$$+ \left(\frac{d_5}{120} - \frac{6 d_6 q}{720}\right) \times T^5$$

$$+ \left(\frac{d_6}{720}\right) \times T^6 \quad . \tag{9}$$

An iterative computer program was written to calculate the thermistor resistance for a given temperature range by means of the power series, Eq. (9), and the defining equation, Eq. (1). This program and the results of these calculations are compiled in Appendix A. The data was computed for a 100° K temperature range centered at q = 283° K. A comparison of the two sets of data showed that the resistance as computed by the power series was about 5 percent low at 233° K and about 100 percent high at 333° K. An error of this magnitude made the power series approach unusable. The accuracy of the power series could be increased by adding more terms to the series, however, this would have made its inclusion in the network equations impractical.

Linearized Thermistor Network

A linearized temperature variable element having the form $R_t = m - nT$, where m and n are positive real constants, would be desirable. A linearized thermistor element would lend itself to

intuitive reasoning in the design of a practical network and would greatly simplify the network equations. A clue to a linearization method for thermistors was obtained from Becker, et. al., (1) in their discussion of temperature compensation of copper devices. If a thermistor and a fixed resistor are paralleled, the resistance of the combination at low temperatures is limited by the fixed resistor and at high temperatures by the thermistor resistance, which is close to zero. This is shown by Fig. 3. The curve of Fig. 3 is essentially linear over some temperature range A. By a proper choice of the thermistor and resistor combination the linear region might be made large enough to span the desired compensation range. The linearized combination could then be used in the design of a compensating circuit so long as the designer remembered that the linear approximation is valid only over a limited temperature range. A trial and error search was instituted using the computer program of Appendix B and the plotting routine of Appendix C. A parallel resistor of value equal to the R_o value of the thermistor gives the largest linear region. As illustrated by Fig. 4, the slope of the linear region of the curve can be varied by using different values of the constant B. The linear temperature range is smaller for larger values of B. A piecewise linear approximation can be made for the whole temperature range. Each of the three major regions can be approximated by a linear section as shown in Fig. 5. Regions 1 and 3 are useful for determining marginal behavior of a network and for determining trial values after a network has been selected. Region 2, approximated by $R_t = m - nT$, is useful in the design of a

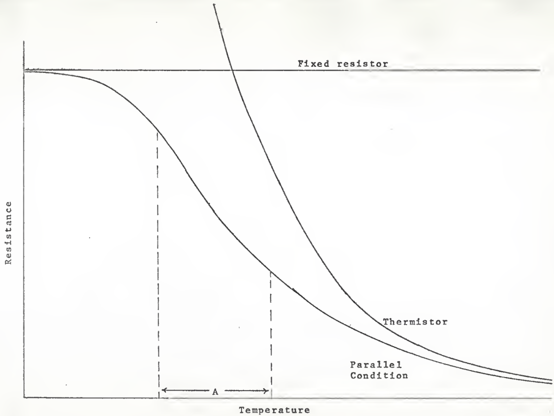


Fig. 3 Temperature-resistance plot for a thermistor and resistor in parallel.

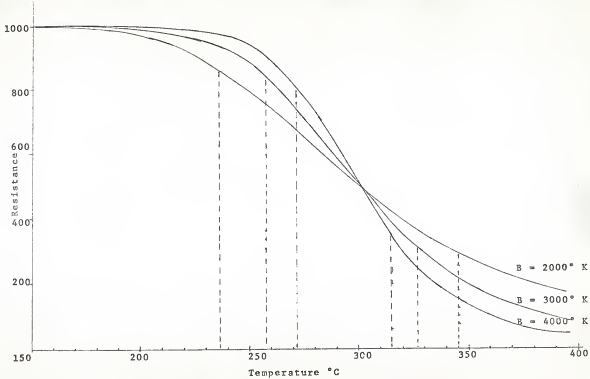


Fig. 4 Temperature-resistance plot showing the effect of varying B.

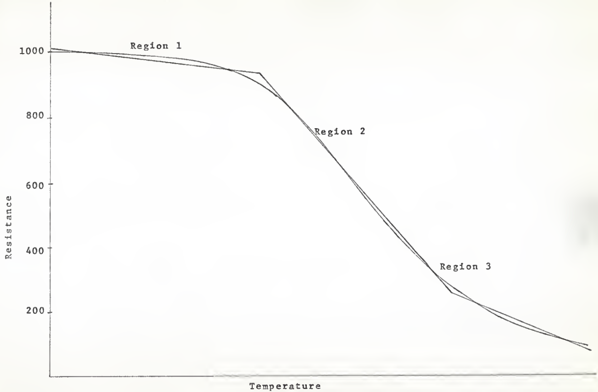


Fig. 5 The piecewise linear approximation.

compensation network. The approximation of region 2 is accurate within about $\pm$ 5 percent, while the high and low temperature regions are accurate within only about $\pm$ 20 percent.

As stated previously, a voltage of the form of Fig. 2 may be obtained by operating on a fixed source voltage with a 2-port network whose transfer function is temperature dependent. In order to cope with many practical compensation problems the output voltage should approximate a polynomial of at least third order in T with two relative extrema. Two operations, amplification and shifting can be performed on the output voltage to bring it into line with the desired control voltage. Amplification is in the normal sense the multiplication of the output voltage by a constant value. Shifting is the addition of a positive or negative voltage in series with the circuit output or amplified output. This is illustrated by the diagram and curves of Fig. 6. In Fig. 6b the curve intersects the zero axis at three locations. These points are known as zeros of the function. The number of zeros and their location determine and help to describe the characteristics of the curve. As shown by Fig. 6c shifting is a versatile tool for altering a voltage-temperature curve. Shifting can change both the number of zeros and their locations. Designing a network which will have one zero and a relative minimum in the positive region as in Fig. 6c is rather difficult, while designing a region as in Fig. 6b is much easier. The idea of shifting makes it possible to design a circuit with an output curve like Fig. 6b and easily convert it to one with an output curve like Fig. 6c.

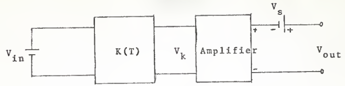


Fig. 6a. Amplifing and shifting circuit for V_k.

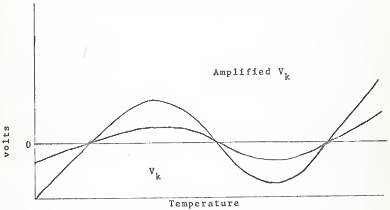


Fig. 6b. Effects of amplification on the voltage-temperature plot.

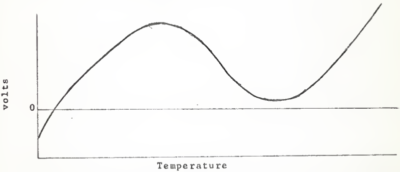


Fig. 6c. Effects of amplification and shifting on the voltage-temperature plot.

Determining a Compensation Network

In an effort to find a network with sufficient generality to compensate various problems with only a change of circuit constants, two network types were examined. The first type relied on the basic voltage divider of Fig. 7, where the circled resistors with enclosed T represent linearized thermistor elements. The linear approximation of the thermistor was used in the analysis of these networks. The transfer functionsof these networks are

$$\frac{V_o}{V_i} = \frac{R}{R+m-nT} \quad \text{and} \quad \frac{V_o}{V_i} = \frac{m-nT}{R+m-nT} \quad . \tag{10}$$

An intuitive view of these transfer functions is given by the voltage transfer ratio vs. temperature plots of Fig. 8. These results seemed to lend promise of obtaining the desired transfer function from a network of this form. The next step was the consideration of a ladder network formed by cascading these voltage divider sections. This network is shown in Fig. 9. The transfer function of the network of Fig. 9 is obtained by solving the matrix equations,

$$\begin{vmatrix} V_i \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} (q+m-nT) & -(m-nT) & 0 \\ -(m-nT) & (r+2m-2nT) & -(m-nT) \\ 0 & -(m-nT) & (s+2m-2nT) \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} \tag{11}$$

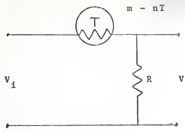


Fig. 7a

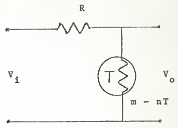


Fig. 7b

Two simple voltage divider networks.

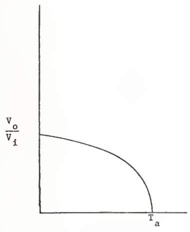


Fig. 8a

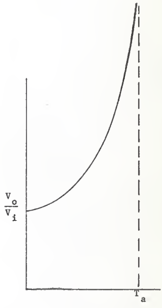


Fig. 8b

Voltage transfer functions for Fig. 7a and 7b.

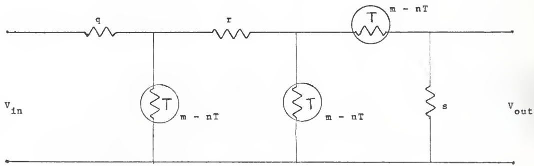


Fig. 9 The three section voltage divider network

and

$$|V_o| = |I_3|\ |s| \quad .$$

The result is

$$\frac{V_o}{V_i} = \frac{a(m-nT)^2}{-(s+2m-2nT)(m-nT)^2+(r+2m-2nT)(s+2m-2nT)(q+m-nT)-(q+m-nT)(m-nT)^2} \quad . (13)$$

Equation (13) is of the general form,

$$\frac{V_o}{V_i} = \frac{e_3T^2 - e_2T + e_1}{-c_4T^3 + c_3T^2 - c_2T + c_1} \quad , \tag{14}$$

where e_i and c_i are constants, depending on the valuea of m, n, r, q, and s. Since the denominator of Eq. (13) is the sum and difference of three third order polynomials in T, the e_i and c_i terms can be either positive or negative depending of the relative values of m, n, r, q, and s. Therefore, no definite conclusion as to the number of zeros and relative extrema of Eq. (14) can be made. Several practical examples were solved and in all cases the tranafer function had only one extremum. Since no practical network constants were found which would yield a more general transfer function this network was abandoned.

The second basic network examined was of the Wheatstone bridge form. The general form of this network is given in Fig. 10. The voltage transfer function of thia network using the linear approximation is

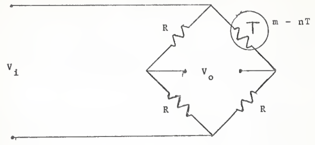


Fig. 10a A single section bridge network with one active element.

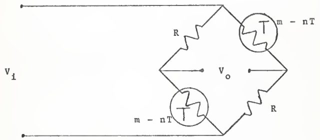


Fig. 10b A single section bridge network with two active elements.

$$\frac{V_o}{V_i} = \frac{R - m + nT}{2R + 2m - 2nT} \quad . \tag{15}$$

At the temperature of the bridge balance point the transfer ratio has the value zero. The balance point may be controlled entirely by the values of the fixed bridge arms. Hence, the location of the transfer function zero may be determined by the values of the fixed bridge arms. A diacussion of bridge theory by Stout (2) indicates that the aensitivity of the bridge to temperature changes can be increased by placing two identical thermistor elements in opposite bridge arms as illustrated by Fig. 10b. This method of increasing sensitivity is employed in the remainder of this report. Two arrangementa of the thermistor elements in the bridge arms are possible. They give the two transfer functions plotted in Fig. 11a and 11b. The curve of Fig. 11a is the negative of the curve in Fig. 11b. The transfer function is not linear, but it is nearly enough to allow the following intuitive reasoning. Two sections with different zero locations may be cascaded. Neglecting loading effects, the combined transfer function should be a function in T having two zeros as shown in Fig. 12. If three sections are cascaded, the output will have three zeros in T with two relative extrema. The three zeros may be located anywhere in the temperature range by changing the balance points of the individual sections. By using the shifting technique, a curve may be produced which has one, two, or three zeros in the range of the linear approximation. It seems that

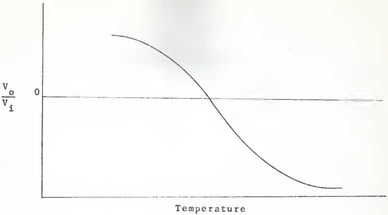


Fig. 11a

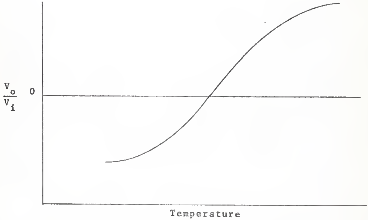


Fig. 12b Two possible transfer functions for the single section bridge.

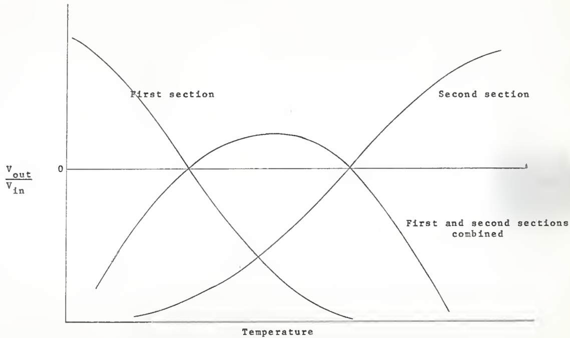


Fig. 12 A probable transfer function for two bridge sections cascaded, neglecting loading.

as many sections as desired could be cascaded, giving a curve having any given number of zeros in T.

Voltage Transfer Function for the Bridge Network

The next step was to verify this intuitive reasoning by computing the actual transfer function for a network consisting of three cascaded bridge sections. The transfer function should include loading effects and use practical values of thermistor constants and bridge arm values. The circuit diagram of a three section bridge is given in Fig. 13. Due to the complexity of the node and loop equations for this network, the topological formulas of Seshu and Reed (3) were used to compute the transfer function. These formulas relate the transfer function to the structure of the circuit. A major advantage of this method is that there is no cancellation of terms as in node or loop analysis.

In order to give the topological formulas, several definitions pertaining to the network graph must be given. A tree of a graph is defined as a connected subgraph containing all the vertices of the original graph but no closed circuits. A 2-tree is a pair of unconnected subgraphs, each subgraph itself being connected, containing all the vertices of the original graph but having one less edge than a tree. A 2-tree is represented by the symbol (2-T). A 2-tree admittance product is the product formed by multiplying the admittances represented by the edges of a 2-tree. A 2-tree admittance product is represented by the symbol W. Consider the generalized graph represented by Fig. 14. A

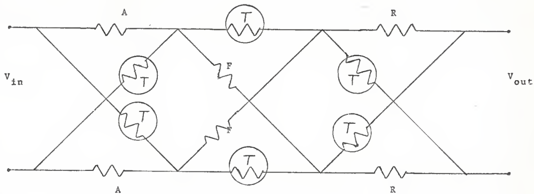


Fig. 13 Circuit diagram for a three section bridge.

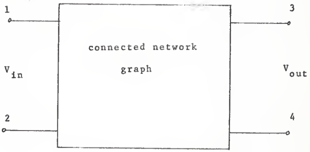


Fig. 14 Generalized two-port network graph.

number of 2-trees of this graph will have vertices 1 and 2 in separate subgraphs. Such 2-trees are represented by the symbology $(2\text{-T})_{1,2}$. Other 2-trees will have 1 and 2 in separate subgraphs and will also have 3 and 4 in separate subgraphs. These are represented by $(2\text{-T})_{13,24}$ and $(2\text{-T})_{14,23}$, where $(2\text{-T})_{13,24}$ denotes those 2-trees such that vertices 1 and 3 are in one connected subgraph and 2 and 4 are in the other subgraph and $(2\text{-T})_{14,23}$ denotes the remaining 2-trees. The corresponding 2-tree admittance products are represented by $W_{1,2}$, $W_{13,24}$, and $W_{14,23}$. The sum of all 2-tree admittance products of a specified class is represented by ΣW. The topological formula for the voltage transfer function of a two port is

$$\frac{V_{out}}{V_{in}} = \frac{\Sigma W_{13,24} - \Sigma W_{14,23}}{\Sigma W_{1,2}} \quad . \tag{16}$$

Formulas for the number of 2-trees of a particular graph and the method of finding those 2-trees are rather complicated. The problem can be reduced to the much easier problem of finding the trees of a similiar graph. If terminals 1 and 2 of Fig. 14 are shorted together and the trees of this graph are found, these trees will be the same as the 2-trees of the original graph having 1 and 2 in separate subgraphs. Likewise if 3 and 4 are shorted, the 2-trees of the original graph having 3 and 4 in separate subgraphs can be found. Those 2-trees represented by $(2\text{-T})_{13,24}$ and $(2\text{-T})_{14,23}$ will be those 2-trees common to both the $(2\text{-T})_{1,2}$ and the $(2\text{-T})_{3,4}$ lists.

Since the method of determining the trees of a large graph is rather involved, it is very helpful to know the actual number of trees before attempting to determine them. For a large graph this is necessary as a check to make certain that all the trees have been found. The number of trees of a graph may be found by evaluating the determinant of the following matrix,

$$[E] = [e_{ij}] \quad , \tag{17}$$

where row i and column i corresponds to the i^{th} vertex of the graph, $i = 1, 2, 3, \cdots, n-1$ (for a graph of n vertices). The element e_{ii} is the number of edges incident at the i^{th} vertex. The element $-e_{ij} = -e_{ji}$, $i \neq j$, is the number of edges connecting the i^{th} and j^{th} verticies. The E matrix for the three section bridge of Fig. 14 with the input shorted is

$$[E] = \begin{vmatrix} 4 & -2 & 0 & 0 & 0 & 0 \\ -2 & 4 & -1 & -1 & 0 & 0 \\ 0 & -1 & 4 & 0 & -1 & -1 \\ 0 & -1 & 0 & 4 & -1 & -1 \\ 0 & 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & -1 & 0 & 2 \end{vmatrix} \quad . \tag{18}$$

The determinant of E is equal to 256. Since the circuit is symmetrical there are 256 $(2\text{-T})_{1,2}$ and 256 $(2\text{-T})_{3,4}$. The trees of the shorted graphs may be found by a method outlined by Percival (4) and illustrated by the example below. Using as an example the graph of Fig. 15a, select two terminals, for example

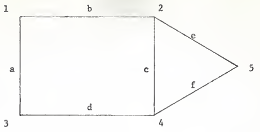


Fig. 15a

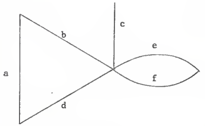


Fig. 15b

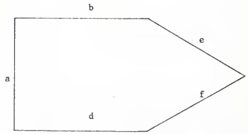


Fig. 15c

Example of finding the trees of a graph by Percival's method.

2 and 4, and construct the two graphs as shown in Fig. 15b and 15c. The trees of Fig. 15b are all of the trees of Fig. 15a which contain the edge c; and the trees of Fig. 15c are all of the trees of Fig. 15a which do not contain the edge c. This process is continued until the trees of the individual graphs so created may be written by inspection. The union of the sets of trees of these individual graphs is equal to the set of all trees of the original graph. The trees of Fig. 15a are as follows: c(a+b+d)(e+f)+abde+abdf+adef+abef+bdef=ace+acf+bce+bcf+dce+dcf+abdf+abde+adef+abef+bdef.

The voltage transfer function of the network of Fig. 13 was found by compiling the lists of all appropriate 2-trees and replacing each edge by the admittance of that arm to obtain the 2-tree admittance products. Then the proper sums of 2-tree admittance products were obtained to satisfy Eq. 16. It seemed possible at this point that by using the linear approximation for the thermistor element and leaving the fixed resistor as unknowns a direct expression in powers of T could be obtained. Then relating this expression to f(t) the fixed resistor unknowns could be determined. However the complexity of this approach due to the large number of terms involved, 256 in the denominator, led the author to abandon this idea. Instead the computer program of Appendix D was written, incorporating the program of Appendix B. This is an iterative program computing the values of all 2-tree admittance products for a given temperature and summing them to obtain the voltage transfer ratio for that temperature. The

value of T is increased and the process is then repeated until the transfer function for the desired temperature range has been found. This program uses actual thermistor constants and bridge arm values. The only factor not included in the program was the possible effects of thermistor self-heating. Previous intuitive reasoning seems to be well substantiated by the results of the program for a sample problem as compiled in Appendix D, Table 1.

The attentuation of the three section bridge is rather high. If in the example of Appendix D, 10 volts were applied to the input of the bridge the maximum output voltage would be 0.09 volts. To be of practical use this voltage would need to be amplified. This could be done by using a low gain, high stability d.c. operational amplifier or by chopping the output and using a higher gain a.c. amplifier. The zero locations of the output are controlled by the fixed bridge arms. Additional flexibility could be achieved by using the voltage shifting principle discussed previously. Therefore the three section cascaded bridge network seems to have sufficient flexibility to be used as a general compensation network for the generation of a variety of temperature compensation voltages.

COMPENSATING A CRYSTAL RESONATOR

The voltage generation method of the last chapter was tested by attempting to generate a control voltage for an actual temperature compensation problem. The problem chosen was the elimination of temperature induced frequency variations from a quartz crystal resonator.

The frequency of a quartz crystal is rather sensitive to variations in temperature. This variation was expressed by Bechmann (5) as a third order function of temperature. The frequency deviation is

$$\frac{\Delta f}{f} = m_1(T-T_o) + m_2(T-T_o)^2 + m_3(T-T_o)^3 \quad , \tag{19}$$

where m_1, m_2, and m_3 are coefficients which depend upon the angle of cut of the crystal, T is the temperature of interest, T_o is some reference temperature, f is the crystal frequency at temperature T_o, and Δf is the deviation of crystal frequency from f. This frequency deviation may range from as little as $\pm$ 10 ppm to $\pm$ 70 ppm or more over the temperature range -40° C to +70° C. Modern communication procedure requires a tolerance of not more than $\pm$ 1 or 2 ppm on resonator frequency. In stationary operations this deviation may be eliminated by placing the crystal in a thermostatically controlled, electrically heated oven. The crystal temperature remains constant and therefore the deviation is eliminated. The power required by the oven and the weight of oven insulation make this method impractical for mobile operations.

A method of compensating a crystal for mobile operations was described by Malik (6). This method will be used with basic alterations. First, the three section bridge will be used for the generation of the compensating voltage. This is a more complex and versatile network and should produce a better compensation. Second, computer techniques were used allowing the constants of the compensation network to be determined more easily.

The crystal frequency may be varied by changing the load capacitance of the crystal. The variation is

$$\frac{\Delta f}{f} = -M + \frac{C_o \times 10^6}{2r(C_o+C_x)} \quad , \tag{20}$$

where M is a constant determined by the value of C_x desired at some frequency f, C_o is the parallel capacitance of the crystal, r is the ratio of series to parallel crystal capacitance, and C_x is the load capacitance. If the capacitance C_x is varied by the proper amount at each temperature, then the frequency deviation will be eliminated. The variable capacitance can be provided by a varactor diode placed across the crystal and biased by the proper d.c. control voltage. The compensation network is shown in Fig. 16. The two varactorsare placed in series to avoid the possibility of their being self-biased by the a.c. signal. The parallel capacitance C_f enables the varactor to be operated in the most desirable part of its characteristic curve. The varactor diode capacitance is

$$C = \frac{K}{V^A} \quad , \tag{21}$$

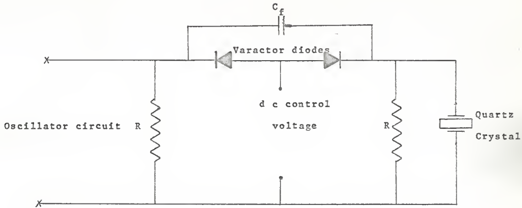


Fig. 16 Crystal compensation circuit.

where K and A are constants and V is the bias voltage. The net capacitance of the two series varactors is

$$C_s = \frac{C}{2} \quad , \tag{22}$$

while the total load capacitance is

$$C_x = C_f + C_s \quad . \tag{23}$$

The solution of Eq. (20), (21), (22), and (23) give the necessary d.c. voltage needed to correct a particular frequency deviation. Starting with a plot of frequency deviation vs. temperature for a crystal, the control voltage necessary to compensate the crystal at each temperature can be computed. This is the voltage which must be supplied by the output of the three section bridge, suitably amplified and shifted. The temperature-voltage relationship necessary for this compensation method may be found by three different methods. The voltage may be found by graphically projecting each point around the curves of Fig. 17. The same data may be obtained by the solution of the equations by the computer program of Appendix E. Both of these methods allow the projection of frequency tolerance curves into voltage tolerance curves. A more accurate method is to actually build the resonator circuit with the varactor diodes and fixed capacitor. Then this whole circuit is placed in a temperature variable oven. The frequency of the resonator is measured by a frequency meter while the control voltage is supplied by a variable d.c. supply voltage. As the temperature is varied a plot of frequency deviation and

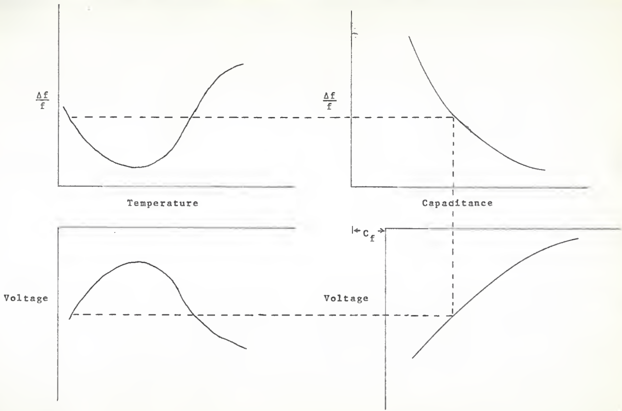


Fig. 17 Graphical Solution of Crystal Equations.

control voltage necessary to correct the frequency is obtained. This method is best, since it relies, not on theory, but on actual circuit operation. Therefore it is generally used for practical problem solutions.

Since a voltage curve obtained by the experimental method was not available, the frequency-temperature data of Table 1, Appendix E was used in conjunction with the computer program of Appendix E to obtain a voltage-temperature curve. This data is compiled in Table 2, Appendix E. The frequency-temperature data of Table 1 is actual data for a Midland one megacycle crystal, however crystal constants were not available. Therefore, reasonable crystal constants were assumed. The voltage-temperature relationship to be obtained as the output of the control network is given in Fig. 18a. The temperature range of this plot is -40 to +70° C. This is a 110° C range, very close to the total linearized range of the thermistor. The output of the program for the three section bridge is in terms of a voltage transfer ratio, while the information desired is in the form of voltage data. The problem can be solved simply by considering the data from the three section bridge program as the voltage output of a bridge energized by a 1 volt input.

Since the bridge output voltage may be shifted at will, the desired voltage curve, Fig. 18a may be likewise shifted as an aid to synthesis. The curve of Fig. 18a has two relative extrema and no zeros. This curve was shifted to have a zero midway between the two extrema as shown in Fig. 18b. As an aid to the location of the other two zeros the piecewise linear approximation and the

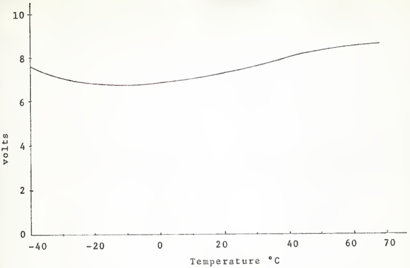


Fig. 18a Desired control circuit output.

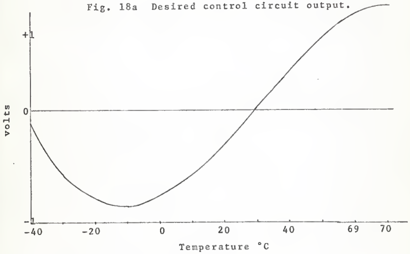


Fig. 18b Desired control circuit output shifted to provide a zero location.

idea of cascading without loading effects were used. The output of a single bridge section containing as its temperature variable element the piecewise linear approximation is given in Fig. 19. If three of these sections were cascaded to obtain the output of Fig. 18b, the zero locations would need to be as shown in Fig. 20. These zero locations are 192° K, 301° K, and 438° K and are used as the first trial in the solution of the transfer function program.

In order to implement the program, the fixed bridge arm values corresponding to the zero locations above must be found. After picking the desired thermistor, the values of the parallel thermistor-resistor combination are computed for values from 150° K to 450° K and compiled as Table 3, Appendix E. To have a zero at 192° K the fixed resistor of the first bridge section must be equal to the parallel thermistor-resistor resistance or 9,964 ohms. Using the data from Table 3, Appendix E the fixed resistor values for the three section were found to be 9,964, 4920, and 410 ohms. The thermistor chosen had constants of B = 3,000° K, T_o = 300° K, and R_o = 10,000 ohms. This data was entered into the bridge transfer function program. The first trial did not yield satisfactory results. The zero locations were amended as seemed necessary and several more trials made. The first concern was to locate the relative extremum at the proper temperature then to adjust the general shape of the curve. When adjusting the extrema locations the higher and lower temperature zeros were moved independently. Moving a zero location by 10 degrees would result

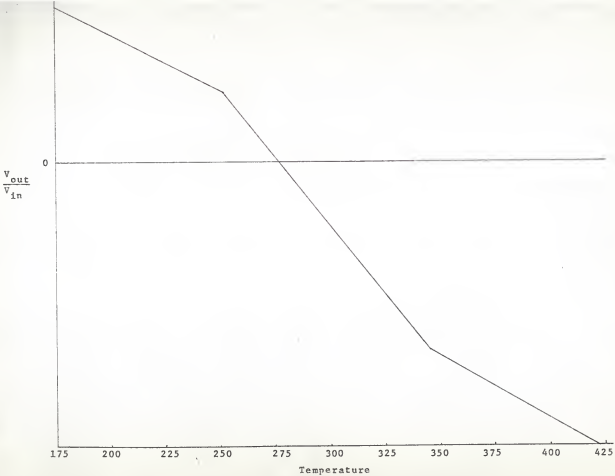


Fig. 19 Piecewise linear approximation of a single bridge section.

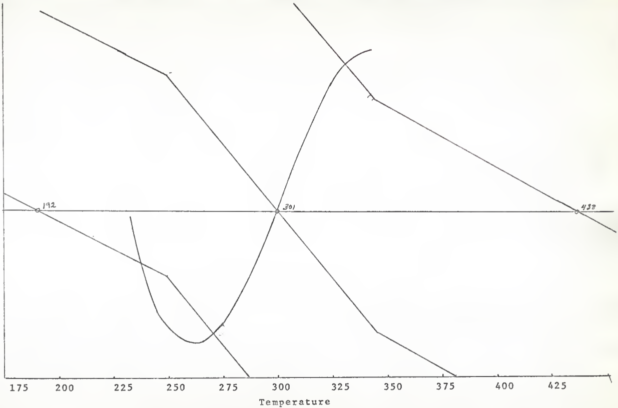


Fig. 20 Determining trial zero locations by using piecewise linear approximation of a single section bridge.

in a 5 degree movement in the location of the adjacent extremum. After the extrema were located, adjusting the curve shape required the simultaneous adjustment of all three zero location in order to preserve the locations of the relative extrema. After several trials a curve of satisfactory shape was obtained except that it was shifted toward the higher temperatures as shown in Fig. 21. The value of T_o was changed to $T_o = 273°$ K. The information of Table 3 was recomputed using the new thermistor data. Appropriate zero location and hence resistor values were chosen. The bridge output voltage curve of this trial was located properly on the temperature axis but the curve was too peaked at the extrema as illustrated by Fig. 22. Therefore the value of B was changed to B = 2,500° K and again the thermistor-resistor parallel resistance was computed. Values of the fixed resistors of 8,860, 3,290, and 845 ohms were chosen for the three bridge sections representing zero locations of 233,296, and 369 degrees Kelvin. The bridge transfer function was computed and seemed to match rather well the voltage curve of Fig. 18a. This transfer function data is compiled in Table 5, Appendix E. The bridge transfer voltage was then amplified and shifted to give the actual control voltage.

The input voltage of the three section bridge was taken to be one volt. This value was chosen to satisfy the current limitations of the thermistor. The maximum current would be about 0.3 milliamperes which should not cause undue thermistor heating. The voltage amplification factor was obtained by

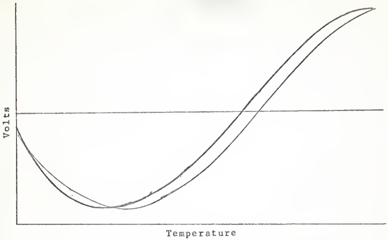


Fig. 21 A control circuit output function showing the effects of too high a value of T_o.

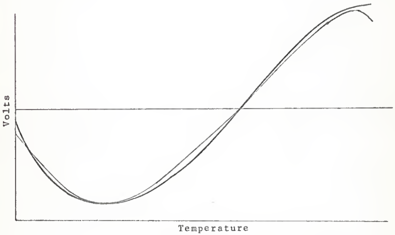


Fig. 22 A control circuit output showing the effects of too high a value of B.

$$\text{Amplification Factor} = \frac{\text{Max desired voltage} - \text{Min desired voltage}}{\text{Max output voltage} - \text{min output voltage}} \quad . \tag{25}$$

Using Eq. (25) the necessary amplification was computed to be

$$\text{Amplification Factor} = \frac{8.5350 - 6.7985}{.01233 - .00725} = 88.678 \quad . \tag{26}$$

The necessary shifting is

$$\text{Shifting Voltage} = \text{Max desired voltage} - (\text{Amplification} \times \text{Max. output voltage}) \quad . \tag{27}$$

It was necessary to reverse the polarity of the bridge output voltage, however in an actual circuit this would only require the reversal of the connection terminals. For this problem,

$$\text{Shifting Voltage} = 8.5350 - 88.678 \times .01233 = 7.4715 \quad . \tag{28}$$

Applying the amplification and shifting of Eq. (26) and (28) to the data of Table 5, Appendix E, the actual control voltage of Table 6, Appendix E was obtained. This information was plotted in Fig. 23 along with the desired control voltage curve and the tolerance voltages which will allow a $\pm$ 1 ppm deviation in resonator frequency. The control voltage is within these tolerances from -35 to +70° C.

A study of allowable tolerances on amplifier gain and on shifting voltage was not attempted. However during the process of determining the proper amplification and shifting it was noted that a variation of up to 6 percent in the amplification factor

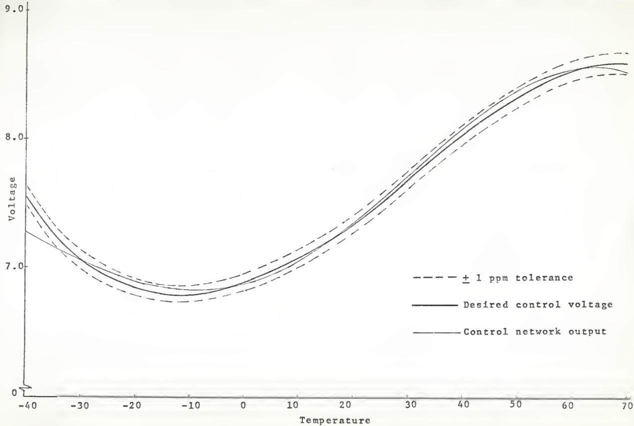
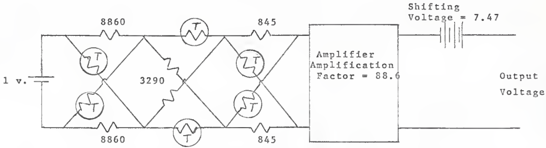


Fig. 23 Control circuit output compared with desired control voltage and tolerance voltages.

would cause only about 1 ppm degradation in the frequency tolerances. Similiarly it was noted that the shifting voltage would need to be held within about 1 percent.

The network shown in Fig. 24 will generate a control voltage which will compensate a Midland one megacycle crystal placed in the circuit of Fig. 17 to within $\pm$ 1 ppm over a 105° C temperature range. Previously described methods of voltage generation (6) had yielded a compensation of approximately $\pm$ 2.5 ppm.



Thermistor constants B = 2500° K T_o = 273° K R_o = 10,000

R_p = 10,000

Fig. 24. Control voltage generating circuit.

CONCLUSION

Certain temperature compensation problems require the generation of a temperature compensation voltage. Various problems and considerations of designing thermistor networks were discussed in this report. The cascaded bridge network was developed and demonstrated to be a versatile method of generating temperature compensated voltages. This network along with the voltage shifting technique was applied to the problem of compensating a crystal resonator for temperature induced frequency variations. The results of theoretical computations using this network show that frequency deviation can be reduced to $\pm$ 1 ppm or less. Computer techniques as listed in the Appendicies were developed to aid in the analysis of this network and in the determination of various network constants.

Several methods of increasing the versatility of the cascaded bridge circuit became evident in the latter stages of this investigation but time was not available to examine them fully. The performance of one stage of the bridge is affected by the resistive load applied to its input and output terminals by the preceeding and following stages. This effect may be varied by altering the impedance level of the various stages. Two transfer functions were computed; in one the impedance level was increased by a factor of ten per section, in the other the impedance level of all stages was the same. The effects of this impedance change were not great, however a proper understanding of loading effects might enable the designer to use them in a limited but beneficial manner.

In this report the same thermistor was used in each section of the bridge, only the fixed arms were changed. This aids the simplicity of the design and would contribute to manufacturing economy. As stated previously the change of the B constant of the thermistor affected the slope of the linearized section. The change of the B or T_o constants of the thermistor in only one section of the bridge might enable the designer to alter the shape of only a portion of the transfer curve.

The zero locations of the transfer function are determined by the fixed arm values of the various bridge sections. In this study the first or low temperature zero location was provided by the first bridge section, the second zero by the second bridge section, and the high temperature zero by the third bridge section. There is no reason that this order could not be reversed or scrambled. For example, the high temperature zero could be provided as a result of the first section, the low temperature zero by the second section, and so forth. Experience gained as a result of this study led the author to believe this technique might have significant effects on the transfer function of the bridge. Although no positive proof was available, this as well as the other factors mentioned would be worthy of further investigation.

ACKNOWLEDGMENTS

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APPENDICES

APPENDIX A

Program 1

```
C  C  PROGRAM FOR APPROXIMATING A THERMISTOR BY A POWER SERIES
C     B=THE BETA CONSTANT OF THE THERMISTOR
C     Q= THE TEMPERATURE AROUND WHICH THE POWER SERIES IS EXPANDED
C     RO= THE RO VALUE OF THE THERMISTOR
C     TO= THE TO CONSTANT OF THE THERMISTOR
      READ,B,Q,RO,TO
      F=EXP(-B/TO)
C     G= THE COMBINATION OF ALL CONSTANT TERMS OF THE THERMISTOR EQUATION
      G=RO*F
      D0=G*EXP(B/Q)
C     D0,D1,...D6=THE THERMISTOR EQUATION AND ITS DERIVATIVES EVALUATED
C        AT THE Q POINT
      R=D0
      D1=-B*R/(Q*Q)
      D2=2.*B*R/(Q*Q*Q)+B*B*R/(Q*Q*Q*Q)
      D3=-6.*B*R/(Q*Q*Q*Q)-6.*B*B*R/(Q**5)-B*B*B*R/(Q**6)
      DA=24.*B*R/(Q**5)+36.*B*B*R/(Q**6)+12.*B**3*R/(Q**7)
      D4=DA+B**4*R/(Q**8)
      DB=120.*B*R/(Q**6)+240.*B*B*R/(Q**7)+120.*B**3*R/(Q**8)
      D5=-DB-20.*B**4*R/(Q**9)-B**5*R/(Q**10)
      DC=720.*B*R/(Q**7)+1800.*B*B*R/(Q**8)+1200.*B*B*B*R/(Q**9)
      D6=DC+300.*B**4*R/(Q**10)+30.*B**5*R/(Q**11)+B**6*R/(Q**12)
C     A0,A1,...A6=THE RESPECTIVE D TERMS DIVIDED BY THE PROPER FACTORIAL
      A0=D0
      A1=D1
      A2=D2/2.
      A3=D3/6.
      A4=D4/24.
      A5=D5/120.
      A6=D6/720.
C     C1,C2,...C7=THE POWERS OF T IN THE EXPANDED POWER SERIES
      C1=A0-A1*Q+A2*Q*Q-A3*Q*Q*Q+A4*Q*Q*Q*Q-A5*Q**5+A6*Q**6
      C2=A1-2.*A2*Q+3.*A3*Q*Q-4.*A4*Q*Q*Q+5.*A5*Q**4-6.*A6*Q**5
      C3=A2-3.*A3*Q+6.*A4*Q*Q-10.*A5*Q*Q*Q+15.*A6*Q*Q*Q*Q
      C4=A3-4.*A4*Q+10.*A5*Q*Q-20.*A6*Q*Q*Q
      C5=A4-5.*A5*Q+15.*A6*Q*Q
      C6=A5-6.*A6*Q
      C7=A6
      T=233.
C     R= THE ACTUAL THERMISTOR RESISTANCE
C     R4= THE POWER SERIES APPROXIMATION TO THE THERMISTOR RESISTANCE
    3 R=G*EXP(B/T)
      R4=C1+C2*T+C3*T*T+C4*T*T*T+C5*T*T*T*T+C6*T**5+C7*T**6
      PUNCH,T,R,R4
      T=T+2.
      IF(T-334.) 3,4,4
    4 CONTINUE
      END
C     TYPICAL INPUT DATA
3000.0    283.0       10000.0     300.0
```

Table 1

Temperature °K	Thermistor	Power Series
2.3300E+02	1.7735E+05	1.6882E+05
2.3500E+02	1.5894E+05	1.5293E+05
2.3700E+02	1.4271E+05	1.3835E+05
2.3900E+02	1.2837E+05	1.2530E+05
2.4100E+02	1.1567E+05	1.1339E+05
2.4300E+02	1.0440E+05	1.0284E+05
2.4500E+02	9.4395E+04	9.3520E+04
2.4700E+02	8.5484E+04	8.4990E+04
2.4900E+02	7.7539E+04	7.7240E+04
2.5100E+02	7.0441E+04	7.0590E+04
2.5300E+02	6.4090E+04	6.4160E+04
2.5500E+02	5.8399E+04	5.8500E+04
2.5700E+02	5.3289E+04	5.3670E+04
2.5900E+02	4.8696E+04	4.8900E+04
2.6100E+02	4.4560E+04	4.4710E+04
2.6300E+02	4.0830E+04	4.0970E+04
2.6500E+02	3.7462E+04	3.7390E+04
2.6700E+02	3.4417E+04	3.4450E+04
2.6900E+02	3.1658E+04	3.2010E+04
2.7100E+02	2.9157E+04	2.9400E+04
2.7300E+02	2.6886E+04	2.7200E+04
2.7500E+02	2.4821E+04	2.5090E+04
2.7700E+02	2.2941E+04	2.3010E+04
2.7900E+02	2.1227E+04	2.1690E+04
2.8100E+02	1.9663E+04	2.0090E+04
2.8300E+02	1.8234E+04	1.8560E+04
2.8500E+02	1.6927E+04	1.7060E+04
2.8700E+02	1.5730E+04	1.5760E+04
2.8900E+02	1.4632E+04	1.5100E+04
2.9100E+02	1.3624E+04	1.4030E+04
2.9300E+02	1.2699E+04	1.3200E+04
2.9500E+02	1.1847E+04	1.2310E+04
2.9700E+02	1.1063E+04	1.1640E+04
2.9900E+02	1.0340E+04	1.0810E+04
3.0100E+02	9.6732E+03	1.0030E+04
3.0300E+02	9.0573E+03	9.2700E+03
3.0500E+02	8.4880E+03	8.8100E+03
3.0700E+02	7.9611E+03	8.2900E+03
3.0900E+02	7.4732E+03	7.7300E+03
3.1100E+02	7.0209E+03	7.3500E+03
3.1300E+02	6.6012E+03	6.9900E+03
3.1500E+02	6.2115E+03	6.7300E+03
3.1700E+02	5.8492E+03	6.3400E+03
3.1900E+02	5.5123E+03	6.5600E+03
3.2100E+02	5.1985E+03	6.1300E+03
3.2300E+02	4.9063E+03	6.3500E+03
3.2500E+02	4.6337E+03	5.8400E+03
3.2700E+02	4.3793E+03	6.2900E+03
3.2900E+02	4.1418E+03	6.1300E+03
3.3100E+02	3.9198E+03	6.3200E+03
3.3300E+02	3.7121E+03	6.2400E+03

APPENDIX B

Program 2

```
C  C   PROGRAM FOR THE RESISTANCE OF A THERMISTOR-RESISTOR IN PARALLEL
C      THERMISTOR CONSTANTSARE B, TO, AND RO
C      RP IS THE PARALLEL RESISTOR, RS IS THE SERIES RESISTOR
     3 READ,B,TO,RO
       READ,RP,RS
C      A,C, AND F ARE CONSTANTS RELATIVE TO THE THERMISTOR VALUE
       C=B/TO
       T=151.
     1 A=B/T
       D=A-C
       F=EXP(D)
C      R= THE THERMISTOR RESISTANCE AND RN= THE COMBINATION RESISTANCE
       R=RO*F
       RN=(R*RP)/(R+RP)+RS
       PUNCH,T,RN,R
       T=T+2.
       IF(T-451.) 1,2,2
     2 CONTINUE
       GO TO 3
       END
C      TYPICAL INPUT DATA
   3000.0      300.0      10000.0
10000.0    0.0
2500.0     273.0      10000.0
10000.0    0.0
```

APPENDIX C

Program 3

```
C   C  PLOTTING ROUTINE FOR FORGO
       DIMENSION P(108),A(36),B(36)
     1 FORMAT(E10.3,2X,108A1)
     2 FORMAT(36A2)
     3 FORMAT(12X,108A1)
     4 FORMAT(2I2)
     9 READ 4,K,J
     8 READ ,AMAX,AMIN
       SCALE=ABS((AMAX-AMIN)/107.0)
       READ 2, (B(I),I=1,K)
     6 DO 15 I=1,108
    15 P(I)=B(1)
       PUNCH 3,(P(I),I=1,108)
       N=2
       L=108
       DO 13 NJK=1,J
       READ, (A(I),I=1,K)
       DO 7 I=N,L
     7 P(I)=0.0
       L=0
       N=108
       DO 14 I=2,K
       M=(A(I)-AMIN)/SCALE+2.0
       IF(L-M) 5,16,16
     5 L=M
    16 IF(N-M) 8,8,17
    17 N=M
     8 IF(P(M)) 11,12,11
    12 P(M)=B(I)
       GO TO 14
    11 P(M)=.14E-36
    14 CONTINUE
    13 PUNCH 1,A(1),(P(I),I=1,L)
       GO TO 9
       END
C      TYPICAL INPUT DATA
 228
9.00    6.000
XXOO
  2.3300E+02  7.2077
  2.4100E+02  7.0475
  2.4900E+02  6.9085
  2.5700E+02  6.8156
  2.6500E+02  6.7935
  2.7300E+02  6.8572
  2.8100E+02  7.0103
  2.8900E+02  7.2404
  2.9700E+02  7.5236
  3.0500E+02  7.8263
```

APPENDIX D

Program 4

```
**MICKELSON
*1006
      DIMENSION Z(256)
    6 FORMAT (F10.5)
    7 FORMAT (2F10.5)
    8 FORMAT (5F10.1)
    9 FORMAT (3I3)
C     N1,N2,AND N3 DEFINE THE TEMPERATURE RANGE
C     A1,F1, AND R1 ARE THE FIXED RESISTORS OF THE BRIDGE SECTIONS
C     BO, TO, RO ARE THERMISTOR CONSTANTS
C     RP AND RS ARE THE LINEARIZING RESISTORS
      READ 9,N1,N2,N3
      READ 8,A1,F1,R1,BO,TO,RO,RP,RS
C     A,D,F,G,R, AND Y ARE ADMITTANCES OF FIXED ARMS
      A=1./A1
      D=A
      F=1./F1
      G=F
      R=1./R1
      Y=R
      CC=BO/TO
      DO 2 M=N1,N2,N3
      AM=M
      AA=BO/AM
      DD=AA-CC
      FF=EXP(DD)
      EE=RO*FF
      U1=(EE*RP)/(EE+RP)+RS
C     B,C,E,H,S, AND T ARE THE ADMITTANCES OF THE THERMISTOR ARMS
C     U1= THE LINEARIZED THERMISTOR
      U2=U1
      U3=U1
      V1=1./U1
      B=V1
      C=V1
      V2=1./U2
      E=V2
      H=V2
      V3=1./U3
      S=V3
      T=V3
C     WA AND WB ARE THE TERMS OF THE TRANSFER FUNCTION NUMERATOR
      WA=A*D*F*G*S*T+B*C*E*H*S*T+A*D*E*H*R*Y+B*C*F*G*R*Y
      WB=A*D*F*G*R*Y+B*C*F*G*S*T+A*D*E*H*S*T+B*C*E*H*R*Y
      W=WA-WB
C     THE Z TERMS ARE THE DENOMINATOR OF THE TRANSFER FUNCTION
      Z(001)=A*D*F*G*R*Y
      Z(002)=A*D*F*G*R*S
```

```
Z(003)=A*D*F*G*T*Y
Z(004)=B*F*G*R*S*T
Z(005)=A*D*F*G*S*T
Z(006)=A*D*G*R*S*Y
Z(007)=A*D*F*R*S*Y
Z(008)=A*D*G*R*T*Y
Z(009)=A*D*F*R*T*Y
Z(010)=A*D*G*R*S*T
Z(011)=A*D*F*R*S*T
Z(012)=A*D*G*H*R*Y
Z(013)=A*D*G*H*R*S
Z(014)=A*D*G*H*T*Y
Z(015)=A*D*G*S*T*Y
Z(016)=A*D*F*S*T*Y
Z(017)=A*D*G*H*S*T
Z(018)=A*D*E*F*R*Y
Z(019)=A*D*E*F*R*S
Z(020)=A*D*E*F*T*Y
Z(021)=A*D*E*F*S*T
Z(022)=A*D*E*R*T*Y
Z(023)=A*D*E*R*S*Y
Z(024)=A*D*H*R*S*Y
Z(025)=A*D*H*R*T*Y
Z(026)=A*D*E*R*S*T
Z(027)=A*D*H*R*S*T
Z(028)=A*D*E*H*R*Y
Z(029)=A*D*E*H*R*S
Z(030)=A*D*E*S*T*Y
Z(031)=A*D*H*S*T*Y
Z(032)=A*D*E*H*S*T
Z(033)=A*F*G*R*S*Y
Z(034)=D*F*G*R*S*Y
Z(035)=A*F*G*R*T*Y
Z(036)=A*F*G*S*T*Y
Z(037)=D*F*G*R*T*Y
Z(038)=A*F*G*R*S*T
Z(039)=A*F*G*H*R*Y
Z(040)=D*F*G*H*R*Y
Z(041)=A*F*G*H*R*S
Z(042)=D*F*G*H*R*S
Z(043)=A*F*G*H*T*Y
Z(044)=D*F*G*H*T*Y
Z(045)=D*F*G*S*T*Y
Z(046)=A*F*G*H*S*T
Z(047)=A*F*H*R*S*Y
Z(048)=D*F*H*R*S*Y
Z(049)=A*F*H*R*T*Y
```

```
Z(050)=D*F*H*R*T*Y
Z(051)=A*F*H*R*S*T
Z(052)=D*F*H*R*S*T
Z(053)=A*F*H*S*T*Y
Z(054)=D*F*H*S*T*Y
Z(055)=A*B*F*G*R*Y
Z(056)=A*E*F*G*R*Y
Z(057)=D*E*F*G*R*Y
Z(058)=A*B*F*G*R*S
Z(059)=A*E*F*G*R*S
Z(060)=A*B*F*G*T*Y
Z(061)=A*E*F*G*T*Y
Z(062)=D*E*F*G*T*Y
Z(063)=B*F*G*H*S*T
Z(064)=A*B*F*G*S*T
Z(065)=A*E*F*G*S*T
Z(066)=D*E*F*G*S*T
Z(067)=A*B*G*R*S*Y
Z(068)=A*E*G*R*S*Y
Z(069)=D*E*G*R*S*Y
Z(070)=A*B*F*R*S*Y
Z(071)=A*B*G*R*T*Y
Z(072)=A*E*G*R*T*Y
Z(073)=D*E*G*R*T*Y
Z(074)=A*B*F*R*T*Y
Z(075)=A*B*G*R*S*T
Z(076)=A*E*G*R*S*T
Z(077)=D*E*G*R*S*T
Z(078)=A*B*F*R*S*T
Z(079)=A*B*G*H*R*Y
Z(080)=A*E*G*H*R*Y
Z(081)=A*E*F*H*R*Y
Z(082)=D*E*G*H*R*Y
Z(083)=D*E*F*H*R*Y
Z(084)=A*B*G*H*R*S
Z(085)=A*E*G*H*R*S
Z(086)=A*E*F*H*R*S
Z(087)=D*E*F*H*R*S
Z(088)=A*B*G*H*T*Y
Z(089)=A*E*G*H*T*Y
Z(090)=D*E*G*H*T*Y
Z(091)=A*E*F*H*T*Y
Z(092)=D*E*F*H*T*Y
Z(093)=A*B*G*S*T*Y
Z(094)=A*E*G*S*T*Y
Z(095)=D*E*G*S*T*Y
Z(096)=A*B*F*S*T*Y
Z(097)=A*B*G*H*S*T
Z(098)=A*E*G*H*S*T
```

```
Z(099)=D*E*G*H*S*T
Z(100)=A*E*F*H*S*T
Z(101)=D*E*F*H*S*T
Z(102)=A*B*E*F*R*Y
Z(103)=A*B*E*F*R*S
Z(104)=A*B*E*F*T*Y
Z(105)=A*B*E*F*S*T
Z(106)=A*B*E*R*S*Y
Z(107)=A*B*H*R*S*Y
Z(108)=A*E*H*R*S*Y
Z(109)=D*E*H*R*S*Y
Z(110)=A*B*E*R*T*Y
Z(111)=A*B*H*R*T*Y
Z(112)=A*E*H*R*T*Y
Z(113)=D*E*H*R*T*Y
Z(114)=A*B*E*R*S*T
Z(115)=A*B*H*R*S*T
Z(116)=A*E*H*R*S*T
Z(117)=D*E*H*R*S*T
Z(118)=A*B*E*H*R*Y
Z(119)=A*B*E*H*R*S
Z(120)=A*D*E*H*T*Y
Z(121)=A*B*E*H*T*Y
Z(122)=A*B*E*S*T*Y
Z(123)=A*B*H*S*T*Y
Z(124)=A*E*H*S*T*Y
Z(125)=D*E*H*S*T*Y
Z(126)=A*B*E*H*S*T
Z(127)=C*D*F*G*R*Y
Z(128)=C*D*F*G*R*S
Z(129)=C*D*F*G*T*Y
Z(130)=D*F*G*H*S*T
Z(131)=C*D*F*G*S*T
Z(132)=C*D*G*R*S*Y
Z(133)=C*D*F*R*S*Y
Z(134)=C*D*G*R*T*Y
Z(135)=C*D*F*R*T*Y
Z(136)=C*D*G*R*S*T
Z(137)=C*D*F*R*S*T
Z(138)=C*D*G*H*R*Y
Z(139)=C*D*G*H*R*S
Z(140)=C*D*G*H*T*Y
Z(141)=C*D*G*S*T*Y
Z(142)=C*D*F*S*T*Y
Z(143)=C*D*G*H*S*T
Z(144)=C*D*E*F*R*Y
Z(145)=C*D*E*F*R*S
Z(146)=C*D*E*F*T*Y
Z(147)=C*D*E*F*S*T
Z(148)=C*D*E*R*S*Y
Z(149)=B*C*H*R*S*Y
Z(150)=C*D*H*R*S*Y
```

```
Z(151)=C*D*E*R*T*Y
Z(152)=C*D*H*R*T*Y
Z(153)=C*D*E*R*S*T
Z(154)=C*D*H*R*S*T
Z(155)=C*D*E*H*R*Y
Z(156)=C*D*E*H*R*S
Z(157)=C*D*E*H*T*Y
Z(158)=C*D*E*S*T*Y
Z(159)=C*D*H*S*T*Y
Z(160)=C*D*E*H*S*T
Z(161)=B*F*G*R*S*Y
Z(162)=C*F*G*R*S*Y
Z(163)=B*F*G*R*T*Y
Z(164)=C*F*G*R*T*Y
Z(165)=C*F*G*R*S*T
Z(166)=B*F*G*H*R*Y
Z(167)=C*F*G*H*R*Y
Z(168)=B*F*G*H*R*S
Z(169)=C*F*G*H*R*S
Z(170)=B*F*G*H*T*Y
Z(171)=C*F*G*H*T*Y
Z(172)=B*F*G*S*T*Y
Z(173)=C*F*G*S*T*Y
Z(174)=C*F*G*H*S*T
Z(175)=B*F*H*R*S*Y
Z(176)=C*F*H*R*S*Y
Z(177)=B*F*H*R*T*Y
Z(178)=C*F*H*R*T*Y
Z(179)=B*F*H*R*S*T
Z(180)=C*F*H*R*S*T
Z(181)=B*F*H*S*T*Y
Z(182)=C*F*H*S*T*Y
Z(183)=B*C*F*G*R*Y
Z(184)=B*E*F*G*R*Y
Z(185)=C*E*F*G*R*Y
Z(186)=B*C*F*G*R*S
Z(187)=D*E*F*G*R*S
Z(188)=B*E*F*G*R*S
Z(189)=C*E*F*G*R*S
Z(190)=B*C*F*G*T*Y
Z(191)=B*E*F*G*T*Y
Z(192)=C*E*F*G*T*Y
Z(193)=B*C*F*G*S*T
Z(194)=D*F*G*R*S*T
Z(195)=B*E*F*G*S*T
Z(196)=C*E*F*G*S*T
Z(197)=B*C*G*R*S*Y
Z(198)=B*E*G*R*S*Y
Z(199)=C*E*G*R*S*Y
Z(200)=C*B*F*R*S*Y
Z(201)=B*C*G*R*T*Y
Z(202)=B*E*G*R*T*Y
Z(203)=C*E*G*R*T*Y
```

```
Z(204)=B*C*F*R*T*Y
Z(205)=B*C*G*R*S*T
Z(206)=B*E*G*R*S*T
Z(207)=C*E*G*R*S*T
Z(208)=B*C*F*R*S*T
Z(209)=B*C*G*H*R*Y
Z(210)=B*E*G*H*R*Y
Z(211)=C*E*G*H*R*Y
Z(212)=B*E*F*H*R*Y
Z(213)=C*E*F*H*R*Y
Z(214)=B*C*G*H*R*S
Z(215)=D*E*G*H*R*S
Z(216)=B*E*G*H*R*S
Z(217)=C*E*G*H*R*S
Z(218)=B*E*F*H*R*S
Z(219)=C*E*F*H*R*S
Z(220)=B*C*G*H*T*Y
Z(221)=B*E*G*H*T*Y
Z(222)=C*E*G*H*T*Y
Z(223)=B*E*F*H*T*Y
Z(224)=C*E*F*H*T*Y
Z(225)=B*C*G*S*T*Y
Z(226)=B*E*G*S*T*Y
Z(227)=C*E*G*S*T*Y
Z(228)=B*C*F*S*T*Y
Z(229)=B*C*G*H*S*T
Z(230)=B*E*G*H*S*T
Z(231)=C*E*G*H*S*T
Z(232)=B*E*F*H*S*T
Z(233)=C*E*F*H*S*T
Z(234)=B*C*E*F*R*Y
Z(235)=B*C*E*F*R*S
Z(236)=B*C*E*F*T*Y
Z(237)=B*C*E*F*S*T
Z(238)=B*C*E*R*S*Y
Z(239)=B*E*H*R*S*Y
Z(240)=C*E*H*R*S*Y
Z(241)=B*C*E*R*T*Y
Z(242)=C*B*H*R*T*Y
Z(243)=B*E*H*R*T*Y
Z(244)=C*E*H*R*T*Y
Z(245)=B*C*E*R*S*T
Z(246)=B*C*H*R*S*T
Z(247)=B*E*H*R*S*T
Z(248)=C*E*H*R*S*T
Z(249)=B*C*E*H*R*Y
Z(250)=B*C*E*H*R*S
Z(251)=B*C*E*H*T*Y
Z(252)=B*C*E*S*T*Y
Z(253)=C*B*H*S*T*Y
Z(254)=B*E*H*S*T*Y
Z(255)=C*E*H*S*T*Y
Z(256)=B*C*E*H*S*T
```

```
      RD=0.0
      DO 3 N=1,256
C     RD IS THE SUM OF THE DENOMINATOR TERMS
    3 RD=RD+Z(N)
C     X IS THE TRANSFER RATIO
C     AM IS THE TEMPERATURE
      X=W/RD
    2 PUNCH 7,AM,X
      END
C     TYPICAL INPUT DATA
233343002
    8862.0    3293.0      845.0     2500.0      273.0
   10000.0   10000.0        0.0
```

Table 1

Voltage Transfer Function for a Sample 3-Section Bridge

$B = 3{,}000°$ K Zeros at 250° K

$T_o = 300°$ K 275° K

$R_o = 1{,}000$ ohms 300° K

Temperature °K	Voltage Transfer Ratio
225.00000	-.00049
227.00000	-.00046
229.00000	-.00042
231.00000	-.00039
233.00000	-.00035
235.00000	-.00031
237.00000	-.00027
239.00000	-.00023
241.00000	-.00018
243.00000	-.00014
245.00000	-.00010
247.00000	-.00006
249.00000	-.00002
251.00000	.00001
253.00000	.00004
255.00000	.00007
257.00000	.00010
259.00000	.00012
261.00000	.00013
263.00000	.00013
265.00000	.00013
267.00000	.00012
269.00000	.00010
271.00000	.00007
273.00000	.00004
275.00000	-.00000
277.00000	-.00004
279.00000	-.00010
281.00000	-.00015
283.00000	-.00021
285.00000	-.00026
287.00000	-.00030
289.00000	-.00033
291.00000	-.00034
293.00000	-.00033
295.00000	-.00029
297.00000	-.00021
299.00000	-.00009
301.00000	.00008
303.00000	.00032

Table 1 (continued)

305.00000	.00063
307.00000	.00102
309.00000	.00151
311.00000	.00209
313.00000	.00278
315.00000	.00360
317.00000	.00454
319.00000	.00563
321.00000	.00686
323.00000	.00826
325.00000	.00982

APPENDIX E

Program 5

```
C  C  PROGRAM TO COMPUTE THE CONTROL VOLTAGE TO COMPENSATE A CRYSTAL
      DIMENSION DFREQ(60),CX(60),DFR(60),P(60),V(3,60),CD(60)
    3 FORMAT (7F10.1)
    4 FORMAT (2E10.3)
    5 FORMAT (F7.2)
C     TMIN IS THE MINIMUM TEMPERATURE AND TMAX IS THE MAXIMUM TEMPERATURE
C     TD IS THE ITERATION INTERVAL
C     FM AND FR ARE CRYSTAL CONSTANTS
C     FK IS A VARACTOR CONSTANT
C     ER IS THE FREQUENCY TOLERANCE IN PPM
      READ3,TMIN,TMAX,TD,FM,FR,FK,ER
C     CO AND CF ARE CIRCUIT CAPACITANCES
      READ 4,CO,CF
      FUDGE=10.**12
   11 T=TMIN
      K=1
C     DFREQ(K) IS THE CRYSTAL DEVIATION AT A PARTICULAR TEMPERATURE
    9 READ 5,DFREQ(K)
      VA=-ER
      J=1
C     DFR(K) IS THE FREQUENCY PLUS TOLERANCE
    6 DFR(K)=-DFREQ(K)+VA
C     CX(K) IS THE CRYSTAL LOAD CAPACITANCE FOR THAT TEMPERATURE
      CX(K)=-CO+(CO*1000000.)/(2.0*FR*(FM+DFR(K)))
C     CD(K) IS THE VARACTOR CAPACITANCE FOR A PARTICULAR TEMPERATURE
      CD(K)=(CX(K)-CF)*2.0*FUDGE
C     V(J,K) IS THE CONTROL VOLTAGE FOR A PARTICULAR TEMPERATURE AND TOLERANCE
      V(J,K)=(FK/CD(K))**2.1739130
      J=J+1
      VA=VA+ER
      IF(ER-VA) 7,6,6
    7 CONTINUE
      PUNCH,T,(V(J,K),J=1,3)
      T=T+TD
      K=K+1
      IF(TMAX-T) 1,9,9
    1 CONTINUE
      END
C     TYPICAL INPUT DATA
    -40.0     70.0      2.0       324.0     208.0     132.0       1.0
+5.000E-12+5.550E-12
  -4.80
  -2.90
  -1.10
    .40
   1.60
   2.40
   3.40
   4.10
```

Table 1

Temperature °C	Δf/f
-40	-4.80
-38	-2.90
-36	-1.10
-34	+ .40
-32	+1.60
-30	+2.40
-28	+3.40
-26	+4.10
-24	+4.80
-22	+5.40
-20	+5.80
-18	+6.20
-16	+6.50
-14	+6.60
-12	+6.70
-10	+6.60
- 8	+6.50
- 6	+6.40
- 4	+6.00
- 2	+5.60
	+5.20
+ 2	+4.80
+ 4	+4.20
+ 6	+3.60
+ 8	+3.00
+10	+2.40
+12	+1.80
+14	+1.00
+16	+ .20
+18	- .60
+20	-1.40
+22	-2.40
+24	-3.30
+26	-4.30
+28	-5.40
+30	-6.40
+32	-7.45
+34	-8.50
+36	-9.40
+38	-10.30
+40	-11.10
+42	-12.00
+44	-13.00
+46	-13.60
+48	-14.40
+50	-15.00
+52	-15.70
+54	-16.30
+56	-16.70
+58	-17.25
+60	-17.80
+62	-18.00
+64	-18.20
+66	-18.40
+68	-18.60
+70	-18.70

Table 2

C C Temp °C	+1 ppm	Control	-1 ppm
-4.0000E+01	7.5039	7.5740	7.6447
-3.8000E+01	7.3721	7.4412	7.5109
-3.6000E+01	7.2489	7.3171	7.3859
-3.4000E+01	7.1476	7.2150	7.2830
-3.2000E+01	7.0673	7.1341	7.2015
-3.0000E+01	7.0142	7.0806	7.1476
-2.8000E+01	6.9482	7.0142	7.0806
-2.6000E+01	6.9024	6.9680	7.0341
-2.4000E+01	6.8568	6.9220	6.9877
-2.2000E+01	6.8179	6.8828	6.9482
-2.0000E+01	6.7920	6.8568	6.9220
-1.8000E+01	6.7662	6.8308	6.8959
-1.6000E+01	6.7470	6.8114	6.8763
-1.4000E+01	6.7406	6.8049	6.8698
-1.2000E+01	6.7341	6.7985	6.8633
-1.0000E+01	6.7406	6.8049	6.8698
-8.0000	6.7470	6.8114	6.8763
-6.0000	6.7534	6.8179	6.8828
-4.0000	6.7791	6.8438	6.9089
-2.0000	6.8049	6.8698	6.9351
.0000	6.8308	6.8959	6.9614
2.0000	6.8568	6.9220	6.9877
4.0000	6.8959	6.9614	7.0274
6.0000	6.9351	7.0010	7.0673
8.0000	6.9746	7.0407	7.1073
1.0000E+01	7.0142	7.0806	7.1476
1.2000E+01	7.0540	7.1207	7.1880
1.4000E+01	7.1073	7.1745	7.2421
1.6000E+01	7.1610	7.2286	7.2966
1.8000E+01	7.2150	7.2830	7.3514
2.0000E+01	7.2693	7.3377	7.4066
2.2000E+01	7.3377	7.4066	7.4760
2.4000E+01	7.3997	7.4690	7.5389
2.6000E+01	7.4690	7.5389	7.6093
2.8000E+01	7.5459	7.6163	7.6873
3.0000E+01	7.6163	7.6873	7.7588
3.2000E+01	7.6909	7.7624	7.8345
3.4000E+01	7.7660	7.8381	7.9107
3.6000E+01	7.8309	7.9034	7.9766
3.8000E+01	7.8962	7.9692	8.0428
4.0000E+01	7.9546	8.0281	8.1021
4.2000E+01	8.0207	8.0947	8.1693
4.4000E+01	8.0947	8.1693	8.2444
4.6000E+01	8.1394	8.2143	8.2897
4.8000E+01	8.1992	8.2746	8.3505
5.0000E+01	8.2444	8.3201	8.3963
5.2000E+01	8.2973	8.3734	8.4500
5.4000E+01	8.3429	8.4193	8.4963
5.6000E+01	8.3734	8.4500	8.5272
5.8000E+01	8.4154	8.4924	8.5699
6.0000E+01	8.4577	8.5350	8.6128
6.2000E+01	8.4731	8.5505	8.6284
6.4000E+01	8.4885	8.5660	8.6441
6.6000E+01	8.5040	8.5816	8.6598
6.8000E+01	8.5195	8.5972	8.6755
7.0000E+01	8.5272	8.6050	8.6833

Table 3

$B = 3000°\ K \quad T_o = 300°\ K \quad R_o = 10{,}000 \quad R_p = 10{,}000$

Temperature °K	Thermistor-Resistor Combination	Thermistor
1.5100E+02	9.9995E+03	1.9294E+08
1.5300E+02	9.9993E+03	1.4881E+08
1.5500E+02	9.9991E+03	1.1555E+08
1.5700E+02	9.9989E+03	9.0298E+07
1.5900E+02	9.9986E+03	7.1005E+07
1.6100E+02	9.9982E+03	5.6169E+07
1.6300E+02	9.9978E+03	4.4689E+07
1.6500E+02	9.9972E+03	3.5753E+07
1.6700E+02	9.9965E+03	2.8758E+07
1.6900E+02	9.9957E+03	2.3250E+07
1.7100E+02	9.9947E+03	1.8891E+07
1.7300E+02	9.9935E+03	1.5423E+07
1.7500E+02	9.9921E+03	1.2650E+07
1.7700E+02	9.9904E+03	1.0423E+07
1.7900E+02	9.9884E+03	8.6245E+06
1.8100E+02	9.9861E+03	7.1665E+06
1.8300E+02	9.9833E+03	5.9791E+06
1.8500E+02	9.9801E+03	5.0080E+06
1.8700E+02	9.9763E+03	4.2106E+06
1.8900E+02	9.9719E+03	3.5532E+06
1.9100E+02	9.9669E+03	3.0091E+06
1.9300E+02	9.9610E+03	2.5571E+06
1.9500E+02	9.9543E+03	2.1803E+06
1.9700E+02	9.9467E+03	1.8650E+06
1.9900E+02	9.9379E+03	1.6003E+06
2.0100E+02	9.9279E+03	1.3774E+06
2.0300E+02	9.9166E+03	1.1891E+06
2.0500E+02	9.9038E+03	1.0294E+06
2.0700E+02	9.8893E+03	8.9367E+05
2.0900E+02	9.8731E+03	7.7794E+05
2.1100E+02	9.8549E+03	6.7898E+05
2.1300E+02	9.8345E+03	5.9413E+05
2.1500E+02	9.8117E+03	5.2117E+05
2.1700E+02	9.7865E+03	4.5828E+05
2.1900E+02	9.7584E+03	4.0392E+05
2.2100E+02	9.7274E+03	3.5683E+05
2.2300E+02	9.6932E+03	3.1592E+05
2.2500E+02	9.6555E+03	2.8032E+05
2.2700E+02	9.6143E+03	2.4925E+05
2.2900E+02	9.5691E+03	2.2208E+05
2.3100E+02	9.5198E+03	1.9826E+05
2.3300E+02	9.4662E+03	1.7735E+05
2.3500E+02	9.4081E+03	1.5894E+05
2.3700E+02	9.3452E+03	1.4271E+05
2.3900E+02	9.2773E+03	1.2837E+05
2.4100E+02	9.2042E+03	1.1567E+05

Table 3 (continued)

2.4300E+02	9.1259E+03	1.0440E+05
2.4500E+02	9.0421E+03	9.4395E+04
2.4700E+02	8.9527E+03	8.5484E+04
2.4900E+02	8.8576E+03	7.7539E+04
2.5100E+02	8.7569E+03	7.0441E+04
2.5300E+02	8.6503E+03	6.4090E+04
2.5500E+02	8.5380E+03	5.8399E+04
2.5700E+02	8.4200E+03	5.3269E+04
2.5900E+02	8.2963E+03	4.8696E+04
2.6100E+02	8.1672E+03	4.4560E+04
2.6300E+02	8.0327E+03	4.0830E+04
2.6500E+02	7.8931E+03	3.7462E+04
2.6700E+02	7.7486E+03	3.4417E+04
2.6900E+02	7.5995E+03	3.1658E+04
2.7100E+02	7.4462E+03	2.9157E+04
2.7300E+02	7.2889E+03	2.6886E+04
2.7500E+02	7.1281E+03	2.4621E+04
2.7700E+02	6.9642E+03	2.2941E+04
2.7900E+02	6.7976E+03	2.1227E+04
2.8100E+02	6.6288E+03	1.9663E+04
2.8300E+02	6.4582E+03	1.8234E+04
2.8500E+02	6.2862E+03	1.6927E+04
2.8700E+02	6.1134E+03	1.5730E+04
2.8900E+02	5.9402E+03	1.4632E+04
2.9100E+02	5.7671E+03	1.3624E+04
2.9300E+02	5.5944E+03	1.2699E+04
2.9500E+02	5.4227E+03	1.1847E+04
2.9700E+02	5.2523E+03	1.1063E+04
2.9900E+02	5.0836E+03	1.0340E+04
3.0100E+02	4.9170E+03	9.6732E+03
3.0300E+02	4.7527E+03	9.0573E+03
3.0500E+02	4.5911E+03	8.4860E+03
3.0700E+02	4.4324E+03	7.9611E+03
3.0900E+02	4.2769E+03	7.4732E+03
3.1100E+02	4.1249E+03	7.0209E+03
3.1300E+02	3.9763E+03	6.6012E+03
3.1500E+02	3.8315E+03	6.2114E+03
3.1700E+02	3.6905E+03	5.8492E+03
3.1900E+02	3.5535E+03	5.5123E+03
3.2100E+02	3.4204E+03	5.1985E+03
3.2300E+02	3.2914E+03	4.9063E+03
3.2500E+02	3.1665E+03	4.6337E+03
3.2700E+02	3.0456E+03	4.3793E+03
3.2900E+02	2.9288E+03	4.1418E+03
3.3100E+02	2.8160E+03	3.9198E+03
3.3300E+02	2.7072E+03	3.7121E+03
3.3500E+02	2.6023E+03	3.5177E+03
3.3700E+02	2.5013E+03	3.3356E+03
3.3900E+02	2.4041E+03	3.1650E+03
3.4100E+02	2.3106E+03	3.0049E+03
3.4300E+02	2.2207E+03	2.8546E+03
3.4500E+02	2.1343E+03	2.7135E+03
3.4700E+02	2.0514E+03	2.5808E+03
3.4900E+02	1.9718E+03	2.4561E+03
3.5100E+02	1.8954E+03	2.3387E+03

Table 3 (continued)

3.5300E+02	1.8221E+03	2.2281E+03
3.5500E+02	1.7519E+03	2.1240E+03
3.5700E+02	1.6845E+03	2.0258E+03
3.5900E+02	1.6200E+03	1.9331E+03
3.6100E+02	1.5581E+03	1.8457E+03
3.6300E+02	1.4988E+03	1.7631E+03
3.6500E+02	1.4420E+03	1.6850E+03
3.6700E+02	1.3876E+03	1.6112E+03
3.6900E+02	1.3355E+03	1.5414E+03
3.7100E+02	1.2856E+03	1.4753E+03
3.7300E+02	1.2378E+03	1.4127E+03
3.7500E+02	1.1920E+03	1.3534E+03
3.7700E+02	1.1482E+03	1.2971E+03
3.7900E+02	1.1062E+03	1.2438E+03
3.8100E+02	1.0660E+03	1.1932E+03
3.8300E+02	1.0274E+03	1.1451E+03
3.8500E+02	9.9053E+02	1.0994E+03
3.8700E+02	9.5517E+02	1.0560E+03
3.8900E+02	9.2129E+02	1.0148E+03
3.9100E+02	8.8882E+02	9.7552E+02
3.9300E+02	8.5770E+02	9.3817E+02
3.9500E+02	8.2787E+02	9.0260E+02
3.9700E+02	7.9928E+02	8.6872E+02
3.9900E+02	7.7186E+02	8.3643E+02
4.0100E+02	7.4557E+02	8.0564E+02
4.0300E+02	7.2036E+02	7.7628E+02
4.0500E+02	6.9617E+02	7.4826E+02
4.0700E+02	6.7296E+02	7.2151E+02
4.0900E+02	6.5068E+02	6.9597E+02
4.1100E+02	6.2930E+02	6.7156E+02
4.1300E+02	6.0878E+02	6.4824E+02
4.1500E+02	5.8907E+02	6.2594E+02
4.1700E+02	5.7014E+02	6.0461E+02
4.1900E+02	5.5196E+02	5.8420E+02
4.2100E+02	5.3448E+02	5.6466E+02
4.2300E+02	5.1769E+02	5.4596E+02
4.2500E+02	5.0155E+02	5.2804E+02
4.2700E+02	4.8603E+02	5.1086E+02
4.2900E+02	4.7111E+02	4.9440E+02
4.3100E+02	4.5675E+02	4.7861E+02
4.3300E+02	4.4294E+02	4.6347E+02
4.3500E+02	4.2965E+02	4.4894E+02
4.3700E+02	4.1686E+02	4.3499E+02
4.3900E+02	4.0454E+02	4.2160E+02
4.4100E+02	3.9268E+02	4.0873E+02
4.4300E+02	3.8126E+02	3.9637E+02
4.4500E+02	3.7025E+02	3.8449E+02
4.4700E+02	3.5965E+02	3.7306E+02
4.4900E+02	3.4942E+02	3.6208E+02

Table 4

$B = 2{,}500°\ K \quad T_o = 273°\ K \quad R_o = 10{,}000 \quad R_p = 10{,}000$

Temperature °K	Thermistor-Resistor Combination	Thermistor
1.5100E+02	9.9939E+03	1.6340E+07
1.5300E+02	9.9924E+03	1.3160E+07
1.5500E+02	9.9906E+03	1.0658E+07
1.5700E+02	9.9885E+03	8.6788E+06
1.5900E+02	9.9859E+03	7.1035E+06
1.6100E+02	9.9829E+03	5.8432E+06
1.6300E+02	9.9793E+03	4.8295E+06
1.6500E+02	9.9751E+03	4.0102E+06
1.6700E+02	9.9702E+03	3.3447E+06
1.6900E+02	9.9644E+03	2.8017E+06
1.7100E+02	9.9577E+03	2.3566E+06
1.7300E+02	9.9500E+03	1.9901E+06
1.7500E+02	9.9411E+03	1.6871E+06
1.7700E+02	9.9308E+03	1.4356E+06
1.7900E+02	9.9191E+03	1.2261E+06
1.8100E+02	9.9057E+03	1.0507E+06
1.8300E+02	9.8905E+03	9.0350E+05
1.8500E+02	9.8733E+03	7.7945E+05
1.8700E+02	9.8539E+03	6.7456E+05
1.8900E+02	9.8321E+03	5.8557E+05
1.9100E+02	9.8076E+03	5.0983E+05
1.9300E+02	9.7803E+03	4.4516E+05
1.9500E+02	9.7499E+03	3.8978E+05
1.9700E+02	9.7161E+03	3.4221E+05
1.9900E+02	9.6787E+03	3.0123E+05
2.0100E+02	9.6375E+03	2.6584E+05
2.0300E+02	9.5921E+03	2.3518E+05
2.0500E+02	9.5424E+03	2.0855E+05
2.0700E+02	9.4882E+03	1.8537E+05
2.0900E+02	9.4290E+03	1.6514E+05
2.1100E+02	9.3648E+03	1.4744E+05
2.1300E+02	9.2954E+03	1.3192E+05
2.1500E+02	9.2204E+03	1.1827E+05
2.1700E+02	9.1398E+03	1.0625E+05
2.1900E+02	9.0534E+03	9.5641E+04
2.2100E+02	8.9611E+03	8.6254E+04
2.2300E+02	8.8628E+03	7.7932E+04
2.2500E+02	8.7584E+03	7.0541E+04
2.2700E+02	8.6480E+03	6.3962E+04
2.2900E+02	8.5315E+03	5.8097E+04
2.3100E+02	8.4091E+03	5.2857E+04
2.3300E+02	8.2808E+03	4.8168E+04
2.3500E+02	8.1469E+03	4.3964E+04
2.3700E+02	8.0075E+03	4.0189E+04
2.3900E+02	7.8630E+03	3.6794E+04
2.4100E+02	7.7135E+03	3.3734E+04
2.4300E+02	7.5594E+03	3.0974E+04
2.4500E+02	7.4012E+03	2.8479E+04
2.4700E+02	7.2391E+03	2.6220E+04

Table 4 (continued)

2.4900E+02	7.0737E+03	2.4173E+04
2.5100E+02	6.9054E+03	2.2314E+04
2.5300E+02	6.7347E+03	2.0625E+04
2.5500E+02	6.5620E+03	1.9087E+04
2.5700E+02	6.3879E+03	1.7685E+04
2.5900E+02	6.2128E+03	1.6405E+04
2.6100E+02	6.0373E+03	1.5235E+04
2.6300E+02	5.8618E+03	1.4165E+04
2.6500E+02	5.6868E+03	1.3184E+04
2.6700E+02	5.5127E+03	1.2285E+04
2.6900E+02	5.3399E+03	1.1459E+04
2.7100E+02	5.1689E+03	1.0699E+04
2.7300E+02	5.0000E+03	1.0000E+04
2.7500E+02	4.8336E+03	9.3557E+03
2.7700E+02	4.6699E+03	8.7613E+03
2.7900E+02	4.5092E+03	8.2124E+03
2.8100E+02	4.3519E+03	7.7050E+03
2.8300E+02	4.1980E+03	7.2355E+03
2.8500E+02	4.0478E+03	6.8006E+03
2.8700E+02	3.9014E+03	6.3973E+03
2.8900E+02	3.7590E+03	6.0231E+03
2.9100E+02	3.6206E+03	5.6754E+03
2.9300E+02	3.4863E+03	5.3522E+03
2.9500E+02	3.3561E+03	5.0513E+03
2.9700E+02	3.2300E+03	4.7711E+03
2.9900E+02	3.1082E+03	4.5099E+03
3.0100E+02	2.9904E+03	4.2662E+03
3.0300E+02	2.8768E+03	4.0386E+03
3.0500E+02	2.7672E+03	3.8259E+03
3.0700E+02	2.6616E+03	3.6270E+03
3.0900E+02	2.5599E+03	3.4408E+03
3.1100E+02	2.4621E+03	3.2663E+03
3.1300E+02	2.3680E+03	3.1028E+03
3.1500E+02	2.2776E+03	2.9493E+03
3.1700E+02	2.1907E+03	2.8053E+03
3.1900E+02	2.1073E+03	2.6700E+03
3.2100E+02	2.0273E+03	2.5427E+03
3.2300E+02	1.9504E+03	2.4230E+03
3.2500E+02	1.8767E+03	2.3103E+03
3.2700E+02	1.8061E+03	2.2041E+03
3.2900E+02	1.7383E+03	2.1040E+03
3.3100E+02	1.6733E+03	2.0096E+03
3.3300E+02	1.6111E+03	1.9205E+03
3.3500E+02	1.5514E+03	1.8363E+03
3.3700E+02	1.4943E+03	1.7568E+03
3.3900E+02	1.4395E+03	1.6815E+03
3.4100E+02	1.3870E+03	1.6104E+03
3.4300E+02	1.3367E+03	1.5430E+03
3.4500E+02	1.2885E+03	1.4791E+03

Table 4 (continued)

3.4700E+02	1.2424E+03	1.4186E+03
3.4900E+02	1.1981E+03	1.3612E+03
3.5100E+02	1.1558E+03	1.3068E+03
3.5300E+02	1.1152E+03	1.2551E+03
3.5500E+02	1.0762E+03	1.2060E+03
3.5700E+02	1.0388E+03	1.1584E+03
3.5900E+02	1.0032E+03	1.1150E+03
3.6100E+02	9.6887E+02	1.0728E+03
3.6300E+02	9.3599E+02	1.0326E+03
3.6500E+02	9.0447E+02	9.9441E+02
3.6700E+02	8.7423E+02	9.5797E+02
3.6900E+02	8.4521E+02	9.2325E+02
3.7100E+02	8.1738E+02	8.9014E+02
3.7300E+02	7.9067E+02	8.5855E+02
3.7500E+02	7.6503E+02	8.2840E+02
3.7700E+02	7.4041E+02	7.9962E+02
3.7900E+02	7.1677E+02	7.7212E+02
3.8100E+02	6.9407E+02	7.4584E+02
3.8300E+02	6.7227E+02	7.2072E+02
3.8500E+02	6.5131E+02	6.9669E+02
3.8700E+02	6.3117E+02	6.7370E+02
3.8900E+02	6.1182E+02	6.5169E+02
3.9100E+02	5.9321E+02	6.3061E+02
3.9300E+02	5.7531E+02	6.1042E+02
3.9500E+02	5.5809E+02	5.9108E+02
3.9700E+02	5.4152E+02	5.7253E+02
3.9900E+02	5.2558E+02	5.5474E+02
4.0100E+02	5.1024E+02	5.3767E+02
4.0300E+02	4.9546E+02	5.2129E+02
4.0500E+02	4.8123E+02	5.0556E+02
4.0700E+02	4.6753E+02	4.9046E+02
4.0900E+02	4.5432E+02	4.7594E+02
4.1100E+02	4.4159E+02	4.6200E+02
4.1300E+02	4.2933E+02	4.4859E+02
4.1500E+02	4.1750E+02	4.3569E+02
4.1700E+02	4.0609E+02	4.2328E+02
4.1900E+02	3.9509E+02	4.1134E+02
4.2100E+02	3.8447E+02	3.9984E+02
4.2300E+02	3.7422E+02	3.8877E+02
4.2500E+02	3.6433E+02	3.7811E+02
4.2700E+02	3.5478E+02	3.6783E+02
4.2900E+02	3.4556E+02	3.5793E+02
4.3100E+02	3.3665E+02	3.4836E+02
4.3300E+02	3.2804E+02	3.3917E+02
4.3500E+02	3.1973E+02	3.3029E+02
4.3700E+02	3.1168E+02	3.2171E+02
4.3900E+02	3.0391E+02	3.1344E+02
4.4100E+02	2.9639E+02	3.0544E+02
4.4300E+02	2.8912E+02	2.9773E+02
4.4500E+02	2.8208E+02	2.9027E+02
4.4700E+02	2.7527E+02	2.8306E+02
4.4900E+02	2.6868E+02	2.7610E+02

Table 5

3-Section Bridge Transfer Function

Temperature °K	Trans. Function
233.00000	.00220
235.00000	.00267
237.00000	.00314
239.00000	.00360
241.00000	.00405
243.00000	.00449
245.00000	.00492
247.00000	.00532
249.00000	.00570
251.00000	.00605
253.00000	.00636
255.00000	.00663
257.00000	.00686
259.00000	.00704
261.00000	.00717
263.00000	.00724
265.00000	.00725
267.00000	.00721
269.00000	.00710
271.00000	.00693
273.00000	.00669
275.00000	.00639
277.00000	.00603
279.00000	.00561
281.00000	.00513
283.00000	.00459
285.00000	.00400
287.00000	.00336
289.00000	.00267
291.00000	.00195
293.00000	.00119
295.00000	.00040
297.00000	-.00041
299.00000	-.00124
301.00000	-.00208
303.00000	-.00293
305.00000	-.00378
307.00000	-.00462
309.00000	-.00545
311.00000	-.00625
313.00000	-.00704
315.00000	-.00778
317.00000	-.00849
319.00000	-.00916
321.00000	-.00977
323.00000	-.01033
325.00000	-.01084
327.00000	-.01127
329.00000	-.01164
331.00000	-.01193
333.00000	-.01215
335.00000	-.01228
337.00000	-.01233
339.00000	-.01230
341.00000	-.01217
343.00000	-.01195

Table 6

Bridge Output Amplified and Shifted

Temperature °K	Control Voltage
2.3300E+02	7.2764
2.3500E+02	7.2347
2.3700E+02	7.1930
2.3900E+02	7.1522
2.4100E+02	7.1123
2.4300E+02	7.0733
2.4500E+02	7.0352
2.4700E+02	6.9997
2.4900E+02	6.9660
2.5100E+02	6.9349
2.5300E+02	6.9075
2.5500E+02	6.8835
2.5700E+02	6.8631
2.5900E+02	6.8471
2.6100E+02	6.8356
2.6300E+02	6.8294
2.6500E+02	6.8285
2.6700E+02	6.8321
2.6900E+02	6.8418
2.7100E+02	6.8569
2.7300E+02	6.8782
2.7500E+02	6.9048
2.7700E+02	6.9367
2.7900E+02	6.9740
2.8100E+02	7.0165
2.8300E+02	7.0644
2.8500E+02	7.1168
2.8700E+02	7.1735
2.8900E+02	7.2347
2.9100E+02	7.2986
2.9300E+02	7.3660
2.9500E+02	7.4360
2.9700E+02	7.5079
2.9900E+02	7.5815
3.0100E+02	7.6560
3.0300E+02	7.7314
3.0500E+02	7.8067
3.0700E+02	7.8812
3.0900E+02	7.9548
3.1100E+02	8.0258
3.1300E+02	8.0959
3.1500E+02	8.1615
3.1700E+02	8.2245
3.1900E+02	8.2839
3.2100E+02	8.3380
3.2300E+02	8.3876
3.2500E+02	8.4329
3.2700E+02	8.4710
3.2900E+02	8.5038
3.3100E+02	8.5295
3.3300E+02	8.5490
3.3500E+02	8.5606
3.3700E+02	8.5650
3.3900E+02	8.5624
3.4100E+02	8.5508
3.4300E+02	8.5313

GENERATION OF TEMPERATURE COMPENSATED VOLTAGES

by

ELDON LEE MICKELSON

B. S., Kansas State University, 1965

AN ABSTRACT OF A MASTER'S REPORT

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MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

Certain temperature compensation problems require the generation of a temperature compensated voltage. Compensating a quartz crystal resonator for frequency variations is one such problem. The frequency can be controlled by loading the crystal with a variable capacitance. This capacitance can be supplied by a varactor biased by a temperature variable control voltage.

The low thermistor cost and the large variety of thermistor types available make it an excellent compensation element. The power dissipation constant of a thermistor must be chosen to match similar thermal constants of the compensated device. Thermistor resistance is given by

$$R = R_o e^{B(1/T - 1/T_o)} .$$

A power series representation of this equation was found to be impractical. A fixed resistor was placed in parallel with the thermistor. This combination could then be approximated by a linear expression over a particular temperature range. This linear expression was used in the analysis and design of possible compensation circuits. Two networks, the cascaded bridge and the resistance ladder were examined as possible compensation networks. The ladder network did not have sufficient flexibility. However, the cascaded bridge network was developed and demonstrated to be a versatile method of generating temperature compensated voltages. The transfer function of the bridge network was found by the use of topological formulas. Digital computer programs were used as an aid to the solution of these

formulas and other calculations. Voltage shifting and amplification techniques were used as an aid to the synthesis of temperature compensated voltages. These techniques along with the cascaded bridge network were applied to the problem of compensating a crystal resonator for temperature induced frequency variations. The results of theoretical computations using this network show that frequency deviation can be reduced to ± 1 ppm or less over a temperature range from $-35°$ to $+70°$ centigrade.